Fast Multipole Techniques For The Simulation of Very Large Three Dimensional Electromagnetic Scattering Problems

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Abstract—**There is an increasing demand for algorithms that can calculate very large electromagnetic problems in a fast and efficient way, for example the detailed electric field distribution for indoor communication. In the past, fast multipole techniques have proven very successful at this. This research is about such techniques for three dimensional structures. The algorithms are developed in such a way that they are suitable for parallel computation. Typical multipole algorithms are suited either for the high frequent or low frequent domain. We intend to investigate broadband algorithms, which combine both regions. This opens the door for applications from the optical domain, over microwave ciruits, to EMC problems. At this moment, a versatile and performant high frequency version is already implemented which will be discussed here.**

Keywords— **Computational Electromagnetics, Fast Multipole Method, Method Of Moments**

I. Introduction

We are attempting full-wave simulations of electromagnetic scattering problems at very large three dimensional objects. Full-wave simulation means that we are solving the exact Maxwell's equations. This allows very accurate results, even for very complicated objects. Approximative techniques, like ray tracing, are only valid under certain assumptions which are not fulfilled for an increasing class of applications. However, the full-wave solution of Maxwell's equations is a very complicated problem, requiring significant computer power. We limit ourselves to the simulation of piecewise homogeneous objects. This allows us to only take their boundaries into consideration. We can write the problem in terms of induced (electric and magnetic) equivalent currents on the surfaces of these objects. These currents can be determined through the Method of Moments, a popular computational technique. Once these currents are known the problem is solved, because the scattered fields can easily be derived from these currents. The Method of Moments, while arbitrarily accurate, is rather inefficient. It will lead to a large linear system of equations that needs to be solved, requiring both a large amount of memory and a long calculation time. Using fast multipole methods, the solution of this linear system will be significantly more efficient, in terms of memory and speed. Further improvement is obtained by parallel computation techniques.

II. Method Of Moments

We start from a group of objects and an incoming field. The objects, excited by the incoming field, will cause a scattered field. The total field is the sum of incoming and scattered fields. The solution of this problem starts by writing the problem in terms of equivalent currents at the surface of these objects [1]. These will cause the scattered field. By application of the necessary boundary conditions (which result directly from Maxwell's equations) an integral equation can be derived which contains these currents as unknown functions over the object surfaces. We now proceed by discretising these currents in a finite set of basis functions [2]. The unknowns are now the coefficients of the expansion in basis functions. We will call N the number of basis functions. We then obtain a linear set of equations by testing the integral equation with the same amount of test functions as there are basis functions, such that this leads to a set of N equations in N unknowns. The test functions are chosen identical to the basis functions, leading to what is called a Galerkin Method of Moments. Solution of this linear system gives the coefficients of the expansion and thus an approximation of the induced currents. This approximation can be made arbitrarily accurate by increasing the number of basis functions. There are many ways to solve a linear system of equations, but two distinct classes can be discerned, namely direct and iterative solution techniques. Direct methods lead to $O(N^3)$ calculation time and include Gaussian elimination and LU-decomposition. Iterative solutions require P iterations to obtain a sufficiently accurate solution, whereas each iteration requires $O(N^2)$ time. Therefore the overall complexity of iterative methods is $O(PN^2)$. Generally, however, P increases slower than N, making these iterative methods vital for large systems of equations. The bottleneck is the $O(N^2)$ time required to calculate a matrix-vector product, upon which the iterative methods are based. Various methods exist to accelerate this matrix-vector product, one of them being the Fast Multipole Method. Note that the memory requirements for both classes of methods are $O(N^2)$ as they require storage of the matrix.

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III. Fast Multipole Method

The matrix describing the linear system of equations (resulting from the Method of Moments) is called the impedance matrix and describes the interaction between the basis and test functions. Since there are N test functions and N basis functions this requires N^2 interactions, stored in the $N \times N$ matrix. An analogy can be made with a very inefficient telephone network, where every telephone is connected by a seperate line to every other telephone. Obviously, this would require an enormous amount of cables. In real networks a group of telephones is linked to a telephone central. These telephone centrals are connected to eachother on a higher level. Even more efficient would be to group several nearby centrals. This grouping approach can also be applied to the impedance matrix. The basis and test functions are divided into groups. The interactions (between groups that are sufficiently far away from eachother) are then treated on a higher level. This allows a significant reduction of the calculations. This Fast Multipole Method (FMM) [3] introduces an error on the matrix elements, but this error is controllable and can be made very small (although higher accuracy leads to more memory and longer calculation time). By using the method on multiple levels, we obtain the Multilevel Fast Multipole Algorithm (MLFMA) [3]. This method has an $O(N \log N)$ complexity for memory and calculation time. This means that, for sufficiently large problems, the behaviour is almost linear, being a dramatic improvement over the classical techniques. Mathematically, there is a significant difference between low frequency and high frequency problems. For low frequency problems the geometry is smaller than the wavelength, while for high frequency problems the geometry is of the order of the wavelength or larger. Currently a high frequent fast multipole method has been implemented, which will be demonstrated in a later section.

IV. Parallelisation

While the MLFMA is already much faster and more memory efficient than the straightforward Method of Moments, even larger problems can be tackled by bundling the power of several processors together. Ideally, using four processors would allow the problem to be calculated four times faster. This is, however, not the case, because parallelisation of the algorithms is not trivial and the processors need to communicate with eachother. Increasing the performance of these parallel calculations is being studied at our departement. The results of this research are being used within the framework of the three dimensional FMM. Typical parallel efficiencies are up to 85% on 16 processors.

V. Implementation

A high-frequency MLFMA method was implemented. The program is called Cassandra and is linked with the open source program Nexus [4], which is developed at our departement and handles the FMM framework and the parallelisation. Cassandra supports the three dimensional scattering at piecewise homogeneous objects, either perfect electric conducting (PEC) or dielectric. The objects are allowed to touch and can even be embedded into eachother. This covers most industrial applications. There exists a very limited amount of configurations that allow analytical solution of Maxwell's equations. Comparisons between these results and simulations with Cassandra show that the implementation is highly accurate.

VI. Examples

In this section we will show an example of a scattering simulation. We have a modelled a large passenger airliner and illuminated it with a frontally incoming plane wave. The plane wave will induce currents on the surface of the airliner. These induced surface currents are shown in Figure 1. The problem consists of

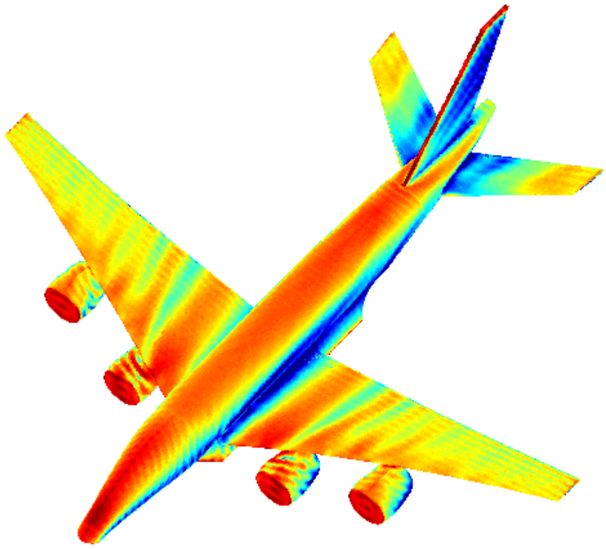

Fig. 1. Induced currents on a large airliner when illuminated by a frontal plane wave

350000 unknowns. Calculation takes about one hour when using 20 processors. The airliner is about 70 meters in length. The wavelength is slightly smaller than 2 meters. Note that this simulation would require 1.8 Terrabyte if solved with a classical Method Of Moments. When solved with Cassandra, every processor requires approximately 300Mb, i.e. 6 Gigabytes in total. Aside from the fact that it would never fit in the memory, straightforwardly executing the matrix-vector product would require a very large amount of time, making it impossible to solve the problem within an acceptable period of time. This example demonstrates the need for efficient methods when large and/or complicated objects are considered.

VII. Conclusion

Full-wave electromagnetic scattering at large and complicated objects requires efficient methods. For the high-frequent case we have implemented a parallel Multilevel Fast Multipole Algorithm which can handle a variety of complicated structures in a fast and memory-efficient way. Future work will concentrate on extending these methods to the entire frequency domain.

References

[1] R.F. Harrington, *Time-harmonic Electromagnetic Fields*, McGraw-Hill Texts In Electrical Engineering, 1961

[2] S. Rao, D. Wilton, A. Glisson, *Electromagnetic Scattering Of Surfaces By Arbitrary Shape*, IEEE Trans. Ant. Prop., volume 30, (3), pp 409-418 (1982)

[3] W.C. Chew, J. Jin, E. Michielssen, J. Song, *Fast And Efficient Methods In Computational Electromagnetics*, Artech House, Inc, 2001.

[4] *http://openfmm.intec.ugent.be*

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